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## References

[1]Heald, M. A., and Wharton, C. B., *Plasma Diagnostics with Microwaves*, Wiley, New York, 1965; reprint, Krieger, Malabar, FL, 1978.

[2]Hutchinson, I. H., *Principles of Plasma Diagnostics*, Cambridge Univ., Press, New York, 1987.

[3]Balwanz, W. W., "Rocket Exhausts and Their Interactions with Electromagnetic Waves," in *AGARDOGRAPH '87, Fluid Dynamics Aspects of Space Flight*, Vol. 2, AGARD, 1966, pp. 307–330.

[4]Gordon, S., and McBride, B. J., "Computer Program for Calculation of Complex Chemical Equilibrium Compositions, Rocket Performance, Incident and Reflected Shocks, and Chapman–Jouget Detonations," NASA SP-273, 1971.

[5]Coleman, H. W., and Steele, W. G., *Experimentation and Uncertainty Analysis for Engineers*, Wiley, New York, 1989.

[6]Blevins, J. A., Frederick, R. A., and Coleman, H. W., "An Assessment of Microwave Measurement Techniques in Rocket Exhaust Applications," AIAA Paper 94-0671, Jan. 1994.

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# Gravity Anchoring for Passive Spacecraft Damping

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## Introduction

MAGNETIC dampers have been used by several gravity-gradient stable spacecraft to provide passive damping. These require no power or communication capability to perform the function of nulling postinjection or deployment rates and to maintain stable steady-state oscillations. Magnetic dampers have been used for passive attitude control in several spacecraft,[1] including the Geodetic Earth Observing Satellite, the Geodynamics Experimental Ocean Satellite, the Gravity Gradient Test Satellite, the Timeation Satellite, and the Long Duration Exposure Facility (LDEF).

A magnetic damper consists of two concentric spheres. The inner sphere contains a permanent magnet that aligns with the Earth's magnetic field lines, and the outer sphere is attached to the spacecraft body. A viscous fluid (for the viscous damper) or a diamagnetic material (for the eddy-current damper) is used between the spheres to produce a damping torque whenever there is relative angular velocity between the two spheres. The damping torque must be large enough to resist relative angular motion, yet small enough to allow the inner magnetic core to maintain orientation with the geomagnetic field.

The major disadvantages of magnetic anchoring are as follows:

1) As the spacecraft orbits the Earth, the local orientation of the Earth's magnetic field changes continually with respect to the local vertical and local horizontal (LVLH). This causes the inner core attitude to vary with respect to the LVLH, which produces undesirable torques on the spacecraft trying to maintain an LVLH attitude. This is especially true for high-inclination orbits passing through the magnetic poles.

2) The damper provides only two-axis damping. Restoring torques exist that align the inner-magnet axis with the Earth's magnetic field lines, but no torque exists to restrict the inner magnet from rotating about the magnetic field vector. For example,[2] in near-equatorial orbits, damping about the pitch axis is small.

In this paper, the use of a new damping methodology, dubbed gravity anchoring, instead of the usual magnetic anchoring, isx proposed to alleviate the disadvantages incurred by the latter. Comparison between the two methodologies, including design considerations and combination of the two methods into a hybrid magnetic–gravity anchoring, is suggested.

## Gravity Anchoring

The idea of gravity anchoring is simple. Why not use a gravity-gradient stable inner core instead of the magnet in the viscous or eddy-current damper discussed earlier? If sufficient gravity-gradient restoring torque and inertia can be provided to the inner core, then gravity anchoring could be realized. The natural frequencies[3] of the linearized gravity-gradient stable dynamics of the inner and outer core must be separated to prevent any phase lock between the inner and outer core, which in turn would prevent damping. An immediate advantage of using the gravity anchor is that this damper does not induce any undesirable steady-state perturbations, since the inner and outer core are both trying to align to the same gravity field. Even though gravity-gradient torques cannot be generated about nadir (yaw axis), gyroscopic coupling[4] could be utilized to provide restoring yaw torques. Thus the gravity anchoring yields three-axis damping. The only remaining question is "Can the inner core be designed to have sufficient inertia and gravity-gradient restoring stiffness?" The answer depends on the spacecraft inertia, size, allowable decay time constant, and pointing requirements. In the following section, the passive damping design problem is addressed, and a comparison between magnetic and gravity anchoring is performed.

## Design Approach

Given an approximate spacecraft configuration, i.e., the inertia properties and the shape, the first task is to identify the maximum anticipated pitch bias angle. This bias angle may be due to biased disturbing solar or aerodynamic torques. Asymmetry and nonzero center-of-pressure–center-of-gravity (CP–CG) offsets can cause these biased disturbing torques. The next task is to determine the minimum required damping to prevent Garber[3] instability. This arises for all gravity-gradient spacecraft in the presence of a constant disturbance torque in the pitch axis. This condition is predicted by the linearized equations of motion when the linearization is performed about the biased pitch, roll, and yaw positions. The pitch bias produces instability in the roll–yaw channels. The pitch bias at which the spacecraft goes unstable is a function of the inertia properties and the damping.[5] Let $K_G$ denote this minimum damping coefficient that avoids Garber instability.

The next task is to define an acceptable decay time from worst-case initial deployment attitude and attitude rates. A good guess can be obtained from simulation, assuming the inner core reference frame to always align with an LVLH frame, i.e., the inner core is perfectly anchored to LVLH. Let this minimum damping coefficient that satisfies the decay-time constraint be denoted by $K_T$. Then the minimum required damping coefficient is $K_D = \max[K_G, K_T]$.

Now the inner core has to be designed so that the restoring magnetic torque on the inner core (for magnetic field anchoring) or the restoring gravity-gradient torque on the inner core (for gravity field anchoring) is larger than the maximum anticipated damping torque. Moreover, the larger the inertia of the inner core, the smaller the effect of the damping torque on its attitude motion. This is particularly important if the restoring torques are small. For magnetic anchoring, large restoring torques can be achieved by increasing the strength of the magnet, which indirectly implies larger mass and size of the passive damper. The disadvantage is the larger steady-state perturbative torques and oscillations. For gravity anchoring, the same can be achieved by increasing the gravity-gradient restoring torques (difference of inertias of the inner core)[4] and the inertia of the inner core, which map directly into larger mass or size of the inner

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core. There are no steady-state perturbative torques using gravity anchoring.

## Numerical Simulation

The LDEF is used as a test satellite to illustrate the idea of gravity anchoring. The three principal-axis moments of inertia of the LDEF are $I_{xx} = 72{,}808$ kg · m$^2$, $I_{yy} = 74{,}299$ kg · m$^2$, and $I_{zz} = 26{,}032$ kg · m$^2$. Note that this configuration is gravity-gradient stable in the absence of any disturbances, since $I_{yy} > I_{xx} > I_{zz}$. Assuming a worst-case 5-cm yaw-axis CP–CG offset, the aerodynamic torque at the lowest altitude of 325 km causes a 2-deg pitch bias.[5] The minimum damping required[5] to avoid Garber instability, $K_G$, for a pitch bias angle of 2 deg is 0.733 N · m · s. However, the following numerical simulations assume no external disturbances. An orbit altitude of 400 km is assumed for the simulation. Initial deployment error is simulated by an initial attitude offset of 15 deg in all three Euler angles of the body axes with respect to LVLH (similar results were obtained with initial maximum[5] tipoff-rate errors of 0.04 deg/s in all three axes). The minimum damping coefficient that satisfies the decay-time constraint, $K_T$, is not prescribed—"LDEF has an acquisition requirement to reach steady-state retrieval conditions within 3 months and it is desirable but not mandatory to reach steady-state experiment pointing conditions within 10 days."[5] Thus a damping coefficient $K_D$ of 0.733 N · m · s is used for the following simulations.

The first simulation assumes an inner core of high inertia given by $I_{xx} = 2900$ kg · m$^2$, $I_{yy} = 3000$ kg · m$^2$, and $I_{zz} = 2800$ kg · m$^2$. Note that the inner core is also gravity-gradient stable, since $I_{yy} > I_{xx} > I_{zz}$. Figure 1 shows that the LDEF yaw angle (lowest inertia axis) is damped to less than 1 deg in about 100 orbits (6 days). Note from Fig. 2 that the damping torque acting on the inner core moves the inner core to about 15 deg in yaw. Similar performance is seen in the pitch and roll axes of the spacecraft and the inner core. The higher the inertia and the gravity-gradient restoring torques of the inner core, the smaller the oscillations of the inner core and the better the damping. The penalty is the larger mass and size of the damper and hence the satellite.

The second simulation assumes a smaller inner core inertia given by $I_{xx} = 290$ kg · m$^2$, $I_{yy} = 300$ kg · m$^2$, and $I_{zz} = 280$ kg · m$^2$.

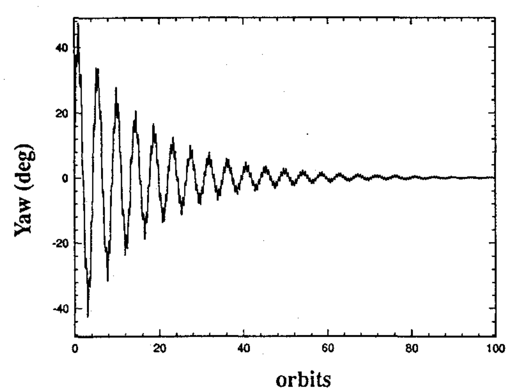


**Fig. 1 LDEF yaw-angle time history (inner core of high inertia).**

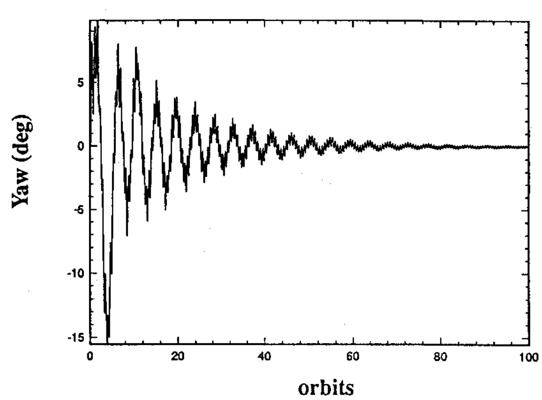


**Fig. 2 Yaw-angle time history of high-inertia inner core.**

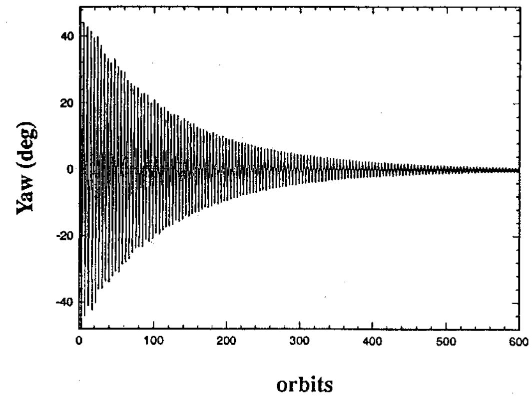


**Fig. 3 LDEF yaw-angle time history (inner core of low inertia).**

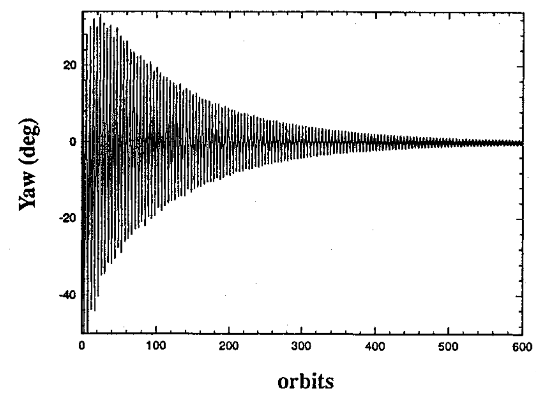


**Fig. 4 Yaw-angle time history of low-inertia inner core.**

Figure 3 shows that the LDEF yaw angle is damped to less than 1 deg in about 600 orbits (36 days). Note from Fig. 4 that the damping torque acting on the smaller inner core moves the inner core to about 50 deg in yaw, which degrades the damping performance. Similar performance is observed in the pitch and roll axes of the spacecraft and the inner core. However, the mass and size of the damping device are much smaller than for the first simulation. Thus, a tradeoff exists between the damping time and the mass and size of the damper.

## Hybrid Design

For magnetic anchoring, the size and mass of the inner core increase as the strength of the magnet is increased to obtain higher restoring torque. The physical dimensions of magnetic dampers are approximately the same for dampers with coefficients less than 3 N · m · s. All have a weight of approximately 10 kg and a diameter under 0.3 m. Thus, it can be tacitly assumed that the size and mass of the magnet are smaller than those of the inner core for obtaining gravity-gradient restoring torques of similar magnitude. If this assumption is violated, then magnetic anchoring should be completely avoided. Otherwise, a hybrid anchoring scheme employing both magnetic and gravity-field anchoring may be the solution.

A gravity-gradient stable electromagnet inner core powered by a discharging battery or a magnet with decreasing strength is envisioned. The high postinjection tipoff rates are damped using magnetic anchoring. As the damping continues, the battery becomes completely discharged and the inner-core restoring torque is provided purely by the gravity-gradient restoring torques. For small rates and angular displacement of the spacecraft achieved after initial deployment rate damping, a small damping coefficient $K_T$ could be chosen, and if $K_G$ is not larger than $K_T$, then the mass and size of the inner gravity-gradient stable core can be kept small. An advantage of the hybrid approach is that the perturbative torques during steady-state spacecraft operation caused by the magnetic anchoring are completely removed and three-axis damping is provided. The battery rate of discharge or the profile of the strength of the magnet has to be determined via preflight simulations for worst-case tipoff rates.

## Conclusions

Magnetic passive damping of gravity-gradient stabilized spacecraft is widely used. The major disadvantages of magnetic anchoring are the perturbative torques exerted on the spacecraft by the inner core aligning to the local magnetic field lines, and that the damper provides only two-axis damping. Gravity anchoring utilizes an inner core that is gravity-gradient stable, and since the inner core and spacecraft align themselves to the same LVLH frame, there are no residual torques during steady-state operations. Moreover, three-axis damping can be achieved. A numerical simulation on the LDEF satellite has been performed to substantiate the idea of gravity anchoring. A hybrid anchoring scheme employing both magnetic and gravity-field anchoring is also suggested in this paper.


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## References


[1]Repass, G. D., Lerner, G. D., Coriell, K. P., and Legg, J. S., Jr., "Geodynamics Experimental Ocean Satellites (GEOS-C) Prelaunch Report," NASA X-580-75-23, Feb. 1975.

[2]Wade, J. W., "Spherical Magnetic Dampers," Lockheed TR LEMSCO-24352, June 1988.

[3]Garber, T. B., "Influence of Constant Disturbing Torques on the Motion of Gravity-Gradient Stabilized Satellites," *AIAA Journal*, Vol. 1, No. 4, 1963, pp. 968, 969.

[4]Wertz, J. R., *Spacecraft Attitude Determination and Control*, D. Reidel, Dordrecht, 1980, pp. 608–612.

[5]Siegel, S. H., and Vishwanath, N. S., "Analysis of the Passive Stabilization of the LDEF," General Electric Final Report, Document No. 78SD4218, Aug. 1977.


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# Spacecraft Expected-Cost Analysis with *k*-out-of-*n* : *G* Subsystems


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## Nomenclature

- $\mathrm{bin}(k-1, n, p)$ = $\sum_{x=0}^{k-1}\binom{n}{x}p^x q^{n-x}$, the cumulative binomial function
- $C$ = cost of the subsystem itself plus expected cost due to subsystem failure
- $c_1$ = loss due to failure of the subsystem
- $c_3$ = cost of a one-module subsystem capable of full output
- $c_4$ = cost of a module in a $k$-out-of-$n$ : $G$ (good) subsystem when $k$ is fixed
- $g(k)$ = function relating cost of the subsystem to number of modules in it
- $k$ = minimum number of good modules for the subsystem to be good
- $n$ = number of modules in the subsystem
- $p$ = probability that a subsystem module is good
- $q$ = probability that a subsystem module fails, $1-p$
- $r$ = reliability of the whole system with respect to hazards other than failure of the subsystem

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## Introduction

IN designing a subsystem for a spacecraft, the design engineer is often faced with a number of options. These options can range from planning an inexpensive subsystem with low reliability to selecting a highly reliable system that would cost much more. How does a design engineer choose between competing subsystems? More particularly, what method can the engineer use to construct models that will take into consideration the various choices offered?

For example, in designing a power subsystem for a spacecraft, the engineer may choose between a power subsystem with 0.960 reliability and a more costly one with 0.995 reliability. When is the increased cost of a more reliable subsystem justified?

High reliability is not necessarily an end in itself, but is desirable in order to reduce the statistically expected cost due to a subsystem failure. However, this may not be the wisest use of funds, since that expected cost is not the only cost involved. The engineer should consider not only the cost of the subsystem but also assess the costs that would occur if the subsystem fails. We therefore minimize the total of the two costs, i.e., the cost of the subsystem plus the expected cost due to subsystem failure, and choose the subsystem with the lowest total.

## *k* out-of-*n* : *G* Subsystems

We will now direct our attention to a specific type of subsystem, called a $k$-out-of-$n$ : $G$ subsystem. Such a subsystem has $n$ modules, of which $k$ are required to be good for the subsystem to be good. As an example consider the situation where the engineer has a certain power requirement for a spacecraft and may meet this requirement by having one large power module ($k = 1$), two smaller modules ($k = 2$), etc. If $k = 4$, then each module yields $\frac{1}{4}$ of the full required power. For example, an $n = 6$ and $k = 4$ subsystem would have 6 modules, each of $\frac{1}{4}$ power, and thus would have the output capability of 1.5 times the required power. The engineer chooses $n$ and $k$. Selection of the different values of $n$ and $k$ results in different subsystems, each with different costs and reliabilities. Thus, we can choose the subsystem ($n$ and $k$) that will minimize the overall expected cost $C$.

The following two models illustrate the principles of the $k$-out-of-$n$ : $G$ subsystem designs. For model 1, the following assumptions are necessary:

1) The probability of failure of any module in the system is not affected by the failure of any other module; i.e., the modules are independent.

2) Each of the modules has the same probability of success.

For model 2 we have the assumptions noted, and we are also free to choose $k$ in our subsystem.

## Model 1

For model 1, we assume that $k$ is fixed and that each module costs $c_4$. Here the engineer may choose only $n$. Now $E\{\text{cost due to subsystem failure}\} = rc_1\Pr\{\text{subsystem failure}\} = rc_1\Pr\{X < k\}$, where $X$, the number of good modules, has a binomial distribution with parameters $n$ and $p$. Since $C$ = (cost of subsystem) + $E\{\text{cost due to subsystem failure}\}$, then

$$C = nc_4 + rc_1\,\mathrm{bin}(k-1; p, n) \qquad (1)$$

The authors have written a program CARRAC (combined analysis of reliability, redundancy, and cost—beta version available) that enables the engineer to select the best subsystems (i.e., the ones with the lowest $C$) and graph $C$ as a function of either $p$ or $c_1$. Since these values are not often known precisely, this graph allows one